Theme Round

Lexington High School

March 23, 2019

Pick Up Lines

1. Anka wants to know if it hurt when her crush fell from the sky. She curls up into a ball and jumps off a 80 meter high building. If she bounces up to $\frac{3}{4}$ of the previous height each bounce, how many times can she bounce while still moving a total distance of less than 300 meters?

2. Alex wants to rearrange the alphabet to put him and his crush next to each other. If he randomly rearranges it, what is the probability that "u" and "i" are next to each other?

3. Jeffrey, being from Tennessee, sees 10s everywhere he looks. If he assigns to each of his 10000 lovers a unique integer ID number from 1 to 10000 how many of them will have the sequence "10" in their ID?

4. Andrew is getting lost in Amy's eyes, or more specifically, her i's: a_i, b_i, c_i, ..., z_i. Let $a_n = \frac{1}{2^n}, b_n = \frac{1}{3^n}, \ldots, z_n = \frac{1}{27^n}$. Additionally, let
$$S = \{(i_1, i_2, \ldots, i_{26}) \mid i_1 \geq 1, i_2, i_3, \ldots, i_{26} \geq 0, i_1, i_2, \ldots, i_{26} \in \mathbb{Z}\}.$$
Find the sum of $a_{i_1} b_{i_2} \ldots z_{i_{26}}$ over all $(i_1, i_2, \ldots, i_{26}) \in S$.

5. Janabel is in love with regular pentagon $ANGEL$ with side length 4 and area x. Find x^2.

Goldilocks Act 1

6. Goldilocks is walking around in the magical forked forest. She has a $\frac{1}{2}$ chance of choosing the correct path at each fork. If she chooses the wrong path more than two times, she gets lost. What is the probability she gets lost if she encounters five forks?

7. Goldilocks is being followed by a squirrel. Whenever the squirrel is 20 ft behind Goldilocks, it runs up until it catches up with her, then stays in place until Goldilocks is 20 ft ahead again. The squirrel runs at a rate of 10 ft/s and Goldilocks walks at a rate of 5 ft/s. If Goldilocks and the squirrel start at the same place, how many seconds will pass before the squirrel catches up to Goldilocks again?

8. Goldilocks walks up to the door, and observes that there are ten locks. She looks under the mattress and finds ten keys. To open the door, every key must be in the correct lock. Moreover, if she puts one key in its correct lock, it cannot be removed anymore. If she tries a random combination of the remaining keys every minute, what is the expected number of minutes until she opens the door?

9. A river runs parallel to Goldilocks' trail. Two congruent circular fields are tangent to the trail, the river, and each other. A third smaller field is externally tangent to the two fields and the river. If the distance between the river and the trail is 200 feet, what is the radius of the smaller field?

10. Goldilocks stumbles upon a large house with an even larger field of flowers. She reads a sign that says, "Three bears in the house, 3^{2019} flowers in the field." If the flowers are arranged in rows of 100, how many flowers will be left over?

Goldilocks Act 2

11. As the bears walk back from their home, they realize they left their fish by the river. The river is a circle centered at their home with radius 6 miles. If they are currently 4 miles away from their home, what is the shortest possible distance, in miles, that they must travel to get to the river and then return home?

12. Goldilocks walks into the bedroom and sees three beds, B_1, B_2, and B_3. Sleepy from her meal, she decides to take a nap in each bed. She slept three times as long in B_2 as B_1 and ten minutes less in B_1 as B_3. If the most time she slept in any single bed was 2019 minutes, how long did she sleep in total?

13. The Bears are on their way back from a stroll. They will return home at a random time between 3 and 4 pm, while Goldilocks will wake up at a random time between 2 and 4 pm. The bears will catch Goldilocks if they arrive home no more than 15 minutes after she wakes up. Find the probability that Goldilocks will escape without being caught.

14. While sleeping in Mama Bear's bed, Goldilocks dreams about different colored sheep. She sees two blue sheep, two red sheep, and one green sheep. How many ways are there to arrange the sheep in a line if no two sheep of the same color can be standing next to each other? Note that sheep of the same color are indistinguishable.

15. A bored and hungry Goldilocks finds an infinite number of raisins in her pocket. On the kitchen table lies an infinite number of bowls of porridge. She labels the bowls, numbering the leftmost bowl 1, the second leftmost bowl 2, and so on. She then distributes her raisins, starting by adding 1 raisin to Bowl 1. Next, if she adds r raisins to Bowl b, then she will add r raisins to Bowl $2b$, and $r+1$ raisins to Bowl $2b+1$. Into how many of the 2^{2019} leftmost bowls of porridge will Goldilocks add exactly 2017 raisins?